Conservative Regridding in ESMF

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Introduction

- The Earth System Modeling Framework[1] [ESMF]
  - Infrastructure and interoperability project started in 2002
  - ESMF is used in NASA, NOAA, DoD, NCAR, and other models
  - Provide standard interfaces to facilitate model coupling and data transfer in large scale earth science applications
- Model coupling often requires remapping data between different grids, or regridding
  - Conservative regridding preserves physical flux fields, for example moisture and heat flux
  - Conservative regridding packages are historically serial, limited to 2D, and file based

Motivation: An Alternative Conservative Method

- Global and local conservation
- Regridding in multiple dimensions
- No requirement to input cell areas
- Multiple integration methods
- Multiple polygon shapes for grid cell representation
- Features for pole treatment and masking
- Parallel and scalable

NOTE:

- This is a reseach project, results are preliminary

- Other ESMF regridding methods use more standard techniques

- ESMF regridding also accepts interpolation weights generated by other applications

How ESMF Regridding Works

- Parallel partitioning - redistribute grids so that overlapping pieces are on the same processor
- Cell search - locate source cell of a destination point
- Interpolation matrix generation - compute and store interpolation weights
- Application of weights - use an optimized sparse matrix multiplication

Options for ESMF Regridding

- Online Regridding
  - Executed using ESMF library calls
  - Supply two ESMF fields $\rightarrow$ build interpolation matrix
  - Apply interpolation matrix using sparse matrix multiplication to interpolate between source and destination fields
- Offline Interpolation Weight Generation Application
  - Separate executable built with ESMF library
  - Supply two netCDF grids $\rightarrow$ interpolation matrix in netCDF file

ESMF Regridding Capabilities

		Online	Offline
2D Polygons	Triangles	√	
	Quadrilaterals	√	√
3D Polygons	Hexahedrons	√	
Regridding	Bilinear	√	
	High Order Patch Recovery[2]	√	√
Masking	Destination	√	√
	Source	√	
	Unmapped points	√	
Pole Options	Full circle average	√	√
	N-point average		√

Conservative Regridding Problem

TASK: Interpolate between two grids on same domain with different:

- Resolution - spacing between grid nodes
- Discretization - arrangement of grid nodes
- Decomposition - division of grid cells among processors

GOAL: Conserve flux quantities during interpolation

- Conserve the global integral of the destination field relative to the source field
- Conserve the integral of the destination field relative to the source field over the region of the local stencil
- Minimize diffusion with repeated remappings

Approach to the Problem

- Use ESMF to compute sparse interpolation weights $\mathcal{I}$
- Adjust $\mathcal{I}$ with a function of integration weights, $\mathcal{F}$

$$A = \mathcal{F}\mathcal{I} \tag{1}$$

- A is the sparse interpolation matrix adjusted for conservation
- Compute a remapped conservative solution

$$u_d = A u_s \tag{2}$$

$$\begin{aligned} u_s &= \text{source field} \\ u_d &= \text{destination field} \end{aligned}$$

- $\mathcal{F}$ requires generation of integration weights...

Integration Weight Generation

- Each cell of the grid has an integration rule over cell domain, Ω

$$\int_{\Omega_j} u = \sum_i \psi_i^j u_i \tag{3}$$

- Where ψ_i^j are local integration weights for node i of cell j

- Sum cell contributions to generate the needed node integration weights, z

$$z_i = \sum_j \psi_i^j \tag{4}$$

L2 Projection

- L2 Projection[3] - Use a standard finite element minimization

$$0 = \int_{\Omega} (u_s - u_d)^2 \quad (5)$$

$$\Rightarrow u_d = M_d^{-1} \mathcal{I}^T M_s u_s \quad (6)$$

where $\mathcal{I}$ is the destination $\rightarrow$ source interpolation matrix, and integration weight matrices

$$M_s = \begin{bmatrix} z_{s,1} & 0 & \dots & 0 \\ 0 & z_{s,2} & 0 & \dots \\ \vdots & & \ddots & \vdots \\ 0 & \dots & 0 & z_{s,m} \end{bmatrix}, M_d^{-1} = \begin{bmatrix} \frac{1}{z_{d,1}} & 0 & \dots & 0 \\ 0 & \frac{1}{z_{d,2}} & 0 & \dots \\ \vdots & & \ddots & \vdots \\ 0 & \dots & 0 & \frac{1}{z_{d,n}} \end{bmatrix}$$

with $m =$ size of u_s and $n =$ size of u_d

Computational Complexity of ESMF Weight Generation

Parallel decomposition	$O(n \log p)$
Cell search	$O(n \log n)$
Interpolation matrix	$O(n)$
L2 Projection	$O(n)$

p = # of processors s = source pts per proc

n = max(d,s) d = destination pts per proc

Test Cases

- Grids used in testing:
  - 2D rectilinear grids
  - 2D spherical grids represented in 3D Cartesian coordinates
    - LL2.5°- latitude longitude 2.5°grid
    - T42 - spherical harmonic grid (spacing $\simeq$ 2.8°)

- Results generated with a prototype in ESMF using Gaussian quadrature

- Accuracy was measured with relative error

  - Interpolation - $(\|u_d - u_{exact}\|_2)/\|u_{exact}\|_2$ (2-norm)
  - Conservation - $|\int u_d - \int u_s / \int u_s|$
  - Integration - $|\int u - \int u_{exact} / \int u_{exact}|$

Integration Weight Test Results

- 2D and 3D grids tested with Gaussian integration weights

Function	2D Relative Integration Error
$F = 1$	10^{-14}
$F = 1 + x + y$	10^{-16}
$F = x^2 + y^2$	10^{-04}
	3D Relative Integration Error
$F = 1$	10^{-04}
$F = Y_2^2$	10^{-02}
$F = Y_{16}^{32}$	10^{-04}

- $Y_2^2 = 2 + \cos^2\theta\cos(2\phi)$ $\quad Y_{16}^{32} = 2 + \sin^{16}(2\theta)\cos(16\phi)$
- Linear and constant functions integrated to roundoff in 2D
- 3D results due to curved cells and undersampling effects

Regridding Results

Bilinear regridding from T42 to LL2.5° before and after conservation

Function	Relative Error	
	Without L2	With L2
Conservation		
$F = 1$	10^{-04}	10^{-15}
$F = Y_2^2$	10^{-04}	10^{-16}
$F = Y_{16}^{32}$	10^{-06}	10^{-15}
Interpolation		
$F = 1$	10^{-17}	10^{-01}
$F = Y_2^2$	10^{-02}	10^{-01}
$F = Y_{16}^{32}$	10^{-02}	10^{-01}

Speculations on Numerical Results

- L2 projection shows good conservation properties

- Interpolation accuracy is sacrificed for global conservation in this formulation

- Possible causes:
  - Integration error due to geometry or undersampling
  - Resolution difference between grids

Regridding with Conservation - ESMF vs. SCRIP

- Spherical Coordinate Remapping and Interp. Package[4] (SCRIP)
- Compare conservative bilinear weights from T42 $\rightarrow$ LL2.5°grids
- Test these weights using SCRIP tester on Y_2^2

	ESMF	SCRIP
Integration	10^{-4} $\surd$	N/A
Interpolation	10^{-2}	10^{-3} $\surd$
Conservation	10^{-14} $\ddot{\smile}$	10^{-4}
Complexity	$O((n/p)\log(n/p))$ Parallel	$O(n \log n)$ Serial

Future work

- Correct integration for spherical geometry
- Test different quadrature rules for integration weight generation
- Compute fractional areas to constrain L2 for local conservation
  - Cell clipping algorithm - locate intersections of source and destination cells
  - Locally conservative L2 projection method
  - Fractional areas are used to construct exchange grids
- Exchange grids - union of two or more parent grids
  - Regridding with exchange grid intermediary is inherently conservative

References

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